ULTIMATE PILE CAPACITY

Because of the non-homogeneity of soil and the unlimited variables that affecting pile behaviour, different methods of approach are, therefore, existed. Among these methods are:-

- a. The static methods,
- b. The load-transfer methods,
- c. The empirical methods
- d. The dynamic methods, and
- e. Field tests methods.

A. STATIC APPROACH

- **Piles in Compression:**
  \[ Q_{ult} = \int_0^L p(x,C_u + K_s \sigma_v \tan \delta) \, dz + A_b (C.N_c + \sigma_v \cdot N_q + 0.5 \gamma \cdot d \cdot N_{\gamma}) - W_p \]  
  \[ Q_{ult} = \sum Q_s + Q_b - W_p \]  
  \[ Q_{all} = \sum Q_s / S.F_t + Q_b / S.F_b - W_p \]  
  \[ Q_{all} = Q_{ult} / S.F \]  

  where:
  
  SF = Safety factor = 2.5 for driven piles, and SF_p = 2.0 or SF_f = 1.0 and SF_b = 3.0 for bored piles.

  \[
  \text{FACTOR OF SAFETY OF SINGLE PILE IN CLAY} \\
  \begin{array}{|c|c|c|}
  \hline
  \text{Type of Pile} & \text{Factor of Safety} & \text{Qall} \\
  \hline
  \text{Driven piles} & S.F. = 2.5 & Q_{all} = Q_p / 2.5 \\
  \hline
  \text{Bored piles} & (i) S.F. = 2.0 & Q_{all} = Q_p / 2.0 \\
  \text{(Take the smaller value)} & (ii) F_1 = 1.0, F_2 = 3.0 & Q_{all} = Q_s / 1.0 + Q_b / 3.0 \\
  \hline
  \end{array}
  \]

- **Piles in Tension:**
  \[ T_{ult} = \sum Q_s + W \]  
  \[ T_{all} = \sum Q_s / S.F_t + W \]  

  where:
  
  \[ Q_{ult} = \text{Ultimate pile capacity}, \quad T_{ult} = \text{Ultimate tension or pullout capacity}, \quad Q_b = \text{End bearing resistance}, \quad Q_s = \text{Frictional resistance}, \quad Q_{all} = \text{Allowable bearing capacity}, \quad W_p = \text{Weight of pile} \]
  
  \[ \approx \text{Weight of removed soil}, \quad d = (\text{Diameter}) \text{ or least dimension of pile}, \quad L = \text{Length of pile}. \]
  
  - End bearing \( \approx 10\% B \) (for driven piles) and \( \approx 30\% B \) (for bored piles and caissons), and
  - Friction \( << 10\% B \).
o For Piles in Clay; Equation (1) will be:
Since for piles in normally consolidated clay the undrained condition is control, \( \phi_u = 0 \) and \( \delta = 0 \)
\[ \therefore k_s \sigma_v \tan \delta = 0 \] and \( c = c_u = q_u / 2 \). Also, for \( \phi_u = 0 \) : \( N_c = 9 \) (from Skempton chart for circular or square), \( N_q = 1.0 \) and \( N_r = 0.0 \). In addition to, the difference between \( [A_b q - W_p] \) is only in \( (\gamma_{conc} - \gamma_{soil}) \) which is very small, that can be neglected, therefore:

\[ Q_p = \int_0^L \pi d(\alpha c_u) dL + A_b (c_u N_c) \] \hspace{1cm} (1-a)

where,
\[ Q_s = \int_0^L \pi d(\alpha c_u) dL = \alpha c_u A_s = C_a A_s ; \]
\[ Q_b = A_b (c_u N_c) , \]
\( \alpha = \) adhesion factor, and
\( C_a = \) adhesion between pile and soil; obtained by one of the following methods:-

1. Tomlinson (1971) \( \alpha - \) method,
2. Meyerhof (1976) \( \beta - \) method,
3. Vijayvergia and Focht (1972) \( \lambda - \) method.

o For Piles in Sand; Equation (1) will be:
Since \( c = 0 \), then \( c . N_c = 0 \) and due to the term \( [A_b (0.5 d r . N_r) - W_p] \) is very small, that can be neglected, therefore:-

\[ Q_p = \int_0^L \pi d(k_s \sigma_v \tan \delta) dL + A_b (q' N_q') \] \hspace{1cm} (1-b)

where,
\[ Q_s = \int_0^L \pi d(k_s \sigma_v(a v g) \tan \delta) dL = \tau_s A_s , \]
\[ Q_b = A_b (q' N_q') , \]
\( q' = \sigma_v ' = \gamma' L = \) Overburden pressure at the base of pile,
\( N_q' = \) Meyerhof's bearing capacity factor for deep foundations,
\( \tau_s = \) Interaction between sand and pile = \( k_s \sigma_v(a v g) \tan \delta \) which should be \( \leq 100 \text{kN/m}^2 \), and
\( A_s = \) Surface area of pile = \( \int_0^L \pi d dL \).
Meyerhof 's (1976) Bearing Capacity Factors (from penetration test data)

- **For** \( \phi_u = 0 \): \( q_{bb} = A_b q' N_q' \leq A_b (9 S_u) \)
- **For** \( \phi > 0 \):
  1. Use \( R_1 = L/B \); obtain \( R_2 = L_c/B \) for the given angle \( \phi \) from Fig.(1),
  2. Enter the curves with \( \phi \):
     - If \( R_1 > 0.5 R_2 \) and \( \phi \leq 30^\circ \) ; obtain factors from the upper \( N'_i \) curves, and
     - If \( R_1 < 0.5 R_2 \) and \( \phi \leq 30^\circ \) ; use a linear ratio between the lower and upper \( N'_i \) curves; from
       \[
       N_q'' = N_q + \frac{R_1}{0.5 R_2} (N'_q - N_q), \quad \text{and} \quad q_b = A_b q' N_q'' \]
     - If \( \phi > 30^\circ \) and depending on \( L/B \); project to the reduced curves shown in the upper right part of Fig.(1) and interpolate as necessary. ((loose or dense sand, soils with varying degrees of compressibility and for overconsolidated (O.C.) clays)).

**Fig.(1): Bearing capacity factors for deep foundations (after Meyerhof, 1976).**

End Bearing \( (Q_b) \) For Driven Piles

- **For Clay Soils (in undrained condition, \( \phi_u = 0 \)):**
  In this case; \( c = S_u \), \( N'c = 9 \) (from Skempton chart for circular or square) and \( N'q = 1.0 \), then the end bearing becomes:
  
  For constant cross-sectional piles:
  \[
  Q_b = A_b (9 S_u) \]
  \[\text{........................................................................................................... (8-a)}\]
For tapered piles (from Skempton, 1966):
\[ Q_b = A_b(9.S_q).\omega \]  .................................................................................................................. (8-b)
where,  \( \omega = 0.80 \) for \( B \leq 1m \), and \( 0.75 \) for \( B > 1m \).

Using Cone Penetration Test (CPT) Results:
\[ Q_b = A_b.q_c ....(\text{units}...of...q_c) \] .................................................................................................................. (8-c)
where,  \( q_c = \) average CPT value in a zone of 8B above to 3B below the pile point.

- **For Sand Soils (\( \phi > 0 \))**:
  If \( L/B < L_c/B \) \( \Rightarrow \)  
  \[ Q_b = A_b.q'.N'_q \]  .................................................................................................................. (9-a)
Provided that \( q'.N'_q \leq 10.7MN/m^2 \) or \( 10700kN/m^2 \).
  If \( L/B \geq L_c/B \) \( \Rightarrow \) 
  \[ Q_b = A_b.q'.N'_q \leq A_b(50.N'_q.\tan \phi) \]  ................................................................ (kN) .................................................................................................................. (9-b)
Provided that \( q'.N'_q \leq 10.7MN/m^2 \) or \( 10700kN/m^2 \).
where,  \( N'_q = \) Meyerhof's bearing capacity factor for deep foundations obtained from Fig.(1).

Using Standard Penetration Test (SPT) Results (Meyerhof's, 1956, 1976):
\[ Q_b = A_b(40.N).L_b / B \leq .400.N.A_b........(kN) \] .................................................................................................................. (9-c)
where,
\( A_b = \) Cross sectional area of pile, \( N = N_{55} = \) Corrected average SPT value in a zone of 8B above to 3B below the pile point, \( L_b = \) Length of the pile embedded in sand, \( B = \) Width or diameter of pile, and \( L_b / B = \) Average depth ratio of the corresponding point into point bearing stratum.

- **For c − \( \phi \) Soils**:
  \[ Q_b = A_b(c.N'_c + \eta q'.N'_q) \]  ..................................................................................................................(10)
where,
\( A_b = \) Cross sectional area of pile, \( c = \) Cohesion, \( q' = \) Effective vertical stress at pile point, \( \eta = 1 \) for all bearing capacity factors except the Vesic (1975) \( N'_f \) factors since \( \eta = (1 + 2k_o)/3 \), \( k_o = \) At rest earth pressure coefficient = \( 1 - \sin\phi \sqrt{OCR} \); where OCR = Overconsolidation ratio, \( N'_c = \) Author's bearing capacity factor for cohesion adjusted for shape and depth, \( N'_q = \) Author's bearing capacity factor adjusted for \( L/B > 1 \) and depends on initial (undisturbed soil) angle of internal friction \( \phi \), \( L = \) Length of pile, and \( B = \) Diameter or least dimension of pile.
**End Bearing ($Q_b$) For Bored Piles**

$$Q_b = \frac{1}{3}(Q_b)_{\text{driven}} \leq A_b(20N'_q \tan \phi) \quad \text{..................................................(11)}$$

**(a) Frictional Resistance ($Q_s$) For Piles In Clay**

- **For driven piles**, the side friction $C_a$ was obtained using one of the following methods:

  $$C_a = \alpha C_u(\text{avg.}) \quad \text{or} \quad C_a = \beta \sigma'_{v(\text{avg.})} \quad \text{or} \quad C_a = \lambda \left[\sigma'_{v(\text{avg.})} + 2C_u(\text{avg.})\right]$$

  Provided that ($C_a \leq 100 \text{kN/m}^2$) due to several factors which affect the adhesion; such as:
  (i) Smear effect that occurs due to drag down of pile during installation,
  (ii) The presence of soft layer overlying a stiff layer, and
  (iii) Shrinkage that occurs in case of stiff clay and leads to separation between pile and soil.

- Annular cracking around the top of pile occurs and therefore ($C_a \leq 0.4C_u$ to a depth = 20 B) provided that ($C_a \leq 100 \text{kN/m}^2$).

  $$C_a \leq 0.4C_u \quad (\text{to a depth} = 20 \text{ B})$$

- **For bored piles**, $Q_s$ can be calculated using either Tomlinson or Meyerhof methods that used for driven piles provided that ($C_a \leq 100 \text{kN/m}^2$).

- **For expanded bored piles**, the following should be taken into consideration:
  (i) If a fissured clay is present at base of pile, then
  \[ C_u(\text{used}) = 0.75C_u(\text{triaxial}) \]
  (ii) If expansion of bored pile exists, neglect the surface area of the shaft side above the expansion to a distance of ($2 \times $ diameter of the shaft); i.e.,
  \[ A_s = \pi.d.(L - 2.d) \]
(1) Tomlinson (1971) $\alpha -$method:

- For $c \neq \phi$ Soils:

\[
Q_s = (\alpha c_u + k_s \sigma_{v(\text{avg.})}, \tan \delta) A_s \tag{12-a}
\]

where,

$\alpha$ = adhesion factor; obtained as before from [Table (2) or Fig.(2-a)]

$\alpha$ -Values of Some Typical Soils (Skempton, 1966).

<table>
<thead>
<tr>
<th>Type of Clay</th>
<th>$\alpha$ - Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Driven Piles</td>
</tr>
<tr>
<td>Londen clay</td>
<td>0.25 - 0.7</td>
</tr>
<tr>
<td>Sensitive clay</td>
<td>1.0</td>
</tr>
<tr>
<td>Highly expansive clay</td>
<td>0.5</td>
</tr>
<tr>
<td>Soft clay</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table (2): Values of adhesion factors for piles driven into stiff to very stiff cohesive soils for design (after Tomlinson, 1971).

<table>
<thead>
<tr>
<th>Case</th>
<th>Soil conditions</th>
<th>Penetration ratio*</th>
<th>Adhesion factor $\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sands or sandy gravels overlying stiff to very stiff cohesive soil</td>
<td>&lt; 20</td>
<td>1.25 Figure (2-a)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt; 20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Soft clays or silts overlying stiff to very stiff cohesive soil</td>
<td>8 &lt; PR \leq 20</td>
<td>0.40 Figure (2-a)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt; 20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Stiff to very stiff cohesive soils without overlying strata</td>
<td>8 &lt; PR \leq 20</td>
<td>0.40 Figure (2-a)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt; 20</td>
<td></td>
</tr>
</tbody>
</table>


\* Penetration Ratio $(PR) = \frac{\text{Depth of Penetration into Cohesive Soil}}{\text{Diameter of Pile}}$

![The $\alpha$ Method](image)

Fig.(2-a): Relationship between soil and adhesion factor (after Tomlinson, 1971, and API, 1984).

- For Clayey Soils $(\phi_u = 0$ in undrained condition):

\[
Q_s = (\alpha c_u) A_s \tag{12-b}
\]

Provided that $[C_a = (\alpha c_u) \leq .100 \text{kN/m}^2]$.
(2) **Meyerhof (1976) β – method:** This method is widely used for soft and medium clays.

\[
Q_s = (k_s \cdot \sigma'_{v(\text{avg})} \cdot \tan \delta)A_s = \beta \cdot \sigma'_{v(\text{avg})} \cdot A_s \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (13)
\]

where,

\[k_s = \text{lateral earth pressure coefficient obtained as follows:}-\]

<table>
<thead>
<tr>
<th></th>
<th>For Driven piles:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>soft to medium clays where ( C_u \leq 100 , kN/m^2 )</strong></td>
<td>[ k_s = (1 - \sin \phi) ] ( k_s = (1 - \sin \phi) \sqrt{OCR} )</td>
</tr>
<tr>
<td><strong>stiff clays where ( C_u &gt; 100 , kN/m^2 )</strong></td>
<td>[ k_s = (1 - \sin \phi) \sqrt{OCR} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>For Bored piles:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>soft to medium clays where ( C_u \leq 100 , kN/m^2 )</strong></td>
<td>[ k_s = (1 - \sin \phi) ] ( k_s = 0.8 )</td>
</tr>
<tr>
<td><strong>stiff clays where ( C_u &gt; 100 , kN/m^2 )</strong></td>
<td>[ k_s = 0.8 ]</td>
</tr>
</tbody>
</table>

\[\sigma'_{\text{avg}} = \text{average effective overburden pressure} = \frac{\gamma' \cdot L}{2},\]

\( \delta = \text{soil to pile friction angle, and} \)
\( \beta = \text{skin friction factor} = k_s \cdot \tan \delta; \text{ obtained from Fig.(2-b).} \)

![Skin friction factor, β, for piles driven into soft and medium clays](image)

(3) **Vijayvergia and Focht (1972) λ – method:** This method is essentialy used for steel piles as well as driven piles in stiff clay.

\[
Q_s = \lambda(\sigma'_{v(\text{avg})} + 2 \cdot C_u(\text{avg})))A_s \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (14)
\]

where,

\[\lambda = \text{dimensionless coefficient; obtained from Fig.(2-c),}\]

\[\sigma'_{\text{avg}} = \text{average effective overburden pressure} = \frac{\gamma' \cdot L}{2}, \text{ and}\]
\[C_u(\text{avg}) = \text{average undrained shear strength.}\]
(4) **Burland method** (1973) *for bored piles*:

\[
Q_s = (s \cdot \alpha \cdot c_u) A_s \tag{15}
\]

where,

- \( s \) = shape factor = 1.0 (for plain shaft-constant cross-sectional pile), and 1.2 (for tapered shaft),
- \( \alpha \) = adhesion factor between soil and pile; obtained as before [from *Table (2) or Fig.(2-a)*].
- \( c_u \) = cohesion.

\[\text{(b) Frictional Resistance (} Q_s \text{) For Piles In Sand}\]

Many methods are available, however, Broom's and Nurdlund's methods are most widely used.

- **Broom's method** (1965); for short piles; i.e. \( \text{PR} = L/B \leq 20 \)

\[
Q_s = \tau_s A_s = (k_s \sigma'_{\text{avg}} \cdot \tan \delta) A_s \tag{16}
\]

where:

- \( \tau_s \) = interaction between sand and pile = \( k_s \sigma'_{\text{avg}} \cdot \tan \delta \) **which should be** \( \leq 100 \text{kN/m}^2 \),
- \( k_s \) = Lateral earth pressure coefficient depends on pile material, angle of contact \( \delta \) between sand and pile, and the relative density of sand; obtained from *Table (3-a)*.

**Table (3-a): Values of \( k_s \) and \( \delta \) for piles in granular soil.**

<table>
<thead>
<tr>
<th>Type of Pile</th>
<th>( \delta^o )</th>
<th>( k_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Loose sand</td>
<td>Dense sand</td>
</tr>
<tr>
<td>Steel</td>
<td>20</td>
<td>0.5</td>
</tr>
<tr>
<td>Concrete</td>
<td>( 3/4 \cdot \phi )</td>
<td>1.0</td>
</tr>
<tr>
<td>Timber</td>
<td>( 2/3 \cdot \phi )</td>
<td>1.5</td>
</tr>
</tbody>
</table>
\( \sigma'_{\text{avg}} = \text{average effective vertical stress on pile segment/segment} \),

\[
\sigma'_{\text{avg}} = \frac{\gamma L}{2} \\
Q_p = Q_s + Q_b
\]

Example (1)

\[
\sigma'_{\text{avg}(1)} = \frac{\gamma_1 L_1}{2} + \frac{\gamma_1 L_1}{2} \\
Q_p = Q_{s1} + Q_{s2} + Q_b
\]

Example (2)

\( \delta = \text{angle of contact between sand and pile, } \phi \text{ must be known and can be obtained from SPT results or the static cone test (CPT) results using Table (3-b)}. \)

**Table (3-b): \( \phi^o \)-values from SPT or CPT results for piles in granular soil.**

<table>
<thead>
<tr>
<th>( k_s )</th>
<th>( \phi^o )</th>
<th>( q_c ) (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low relative density</td>
<td>28 – 30</td>
<td>0 – 5000</td>
</tr>
<tr>
<td>Medium relative density</td>
<td>30 - 36</td>
<td>5000 – 10000</td>
</tr>
<tr>
<td>High relative density</td>
<td>&gt;36</td>
<td>&gt; 10000</td>
</tr>
</tbody>
</table>

- **Nurdlund's method (1965); for layered soil and lengthy piles:** i.e. \( PR = L/B > 20 \)

\[ Q_s = \tau_s A_s \]

For tapered pile:

\[ Q_s = \sum_{d=0}^{L} (k_s \sigma'_{\text{avg}} \cdot \sin \delta)_C d \cdot d \]  \( \text{(17-a)} \)

For uniform pile:

\[ Q_s = (k_s \sigma'_{\text{avg}} \cdot \sin \delta)_A s \]  \( \text{(17-b)} \)

where,

\[ \tau_s = k_s \sigma'_{\text{avg}} \cdot \sin \delta \text{ which should be } \leq 0.107 \text{MN/m}^2 \text{ or } 10700 \text{kN/m}^2, \]

\( k_s, \sigma'_{\text{avg}}, \sin \delta \) as previously determined.

\( C_d = \text{minimum perimeter of element at depth } d. \)

\( d = \text{depth of element (length of segment)}. \)
Steps of Solution by Nurdlund’s Method

1. Divide the pile into several segments with respect to soil profile or property.

2. Calculate $V(m^3/m)$; the volume of the pile material for each segment per unit length.

3. Obtain $(\delta/\phi)$ using Fig.(3) with $V(m^3/m) = ?$ and the type of the pile material (concrete, steel,……etc.).

4. Find $(\phi)$ from penetration tests; either Dynamic Penetration Test (S.P.T.) or Static Cone Test (C.P.T.): $N \rightarrow \phi$, or $q_c \rightarrow \phi$

5. Find $(k_s)$ using Fig.(4) and assuming that $\delta = \phi$.

6. Find $(k_s)_{\text{correction factor}}$ using Fig.(5) when $\delta/\phi \neq 1.0$.

7. $Q_s = [k_s \cdot \sigma_V^{\text{avg}} \cdot \sin(\delta)] \cdot A_s = \tau_s \cdot A_s \leq 10700$ kN/m$^2$; for successive sections, the process is repeated, such that: $\sum Q_s = Q_{s1} + Q_{s2} + ........$

8. $Q_b = (q' \cdot N_q') \cdot A_b \leq 10.7 MN/m^2$...or...10700 kN/m$^2$

9. $Q_p = \sum Q_s + Q_b$

Fig.(3): Relationship between $\delta/\phi$ and volume displacement of driven pile (after Nordlund, 1965).
Fig.(4): Values of coefficient of lateral earth pressure $k_s$

(after Nordlund, 1965).

Figure (5): Correction factor for $k_s$ (after Nordlund, 1965).
PILE FOUNDATIONS GROUP

Pile foundation is rarely consists of a single pile. In general, piles are used in groups. The minimum number of piles in each group should be not less than two or three piles. The Chicago Building Code states that "A column or pier supported by piles shall rest on not less than three piles". Fig.(6) presents some typical pile clusters for illustrative purposes only, since the designer must make up the group geometry to satisfy any given problem.

Fig.(6): Typical pile-group patterns: (a) for isolated pile caps; (b) for foundation walls.
SPACING OF PILES

When several piles are clustered, the soil pressures produced from either side friction or point bearing will overlap as idealized in Fig.(7) depending upon piles loads and their number and spacings. If these pressures sufficiently are large, the soil will fail in shear or excessive settlement. However, large spacings between piles are often impractical since a pile cap is to be cast over the pile group for the column base and/ or to spread the load to the several piles in the group.

Suggested minimum center-to-center piles spacing by several building codes are as follows:

<table>
<thead>
<tr>
<th>Pile type</th>
<th>NBC, 1976</th>
<th>BOCA, 1993</th>
<th>Chicago, 1994</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction</td>
<td>2D or 1.75H &gt; 760 mm</td>
<td>2D or 1.75H &gt; 760 mm</td>
<td>2D or 2H &gt; 760 mm</td>
</tr>
<tr>
<td>Point bearing</td>
<td>2D or 1.75H &gt; 610 mm</td>
<td>2D or 1.75H &gt; 610 mm</td>
<td></td>
</tr>
</tbody>
</table>

Fig.(7): Stresses surrounding a friction pile and the summing effects of a pile group.

where: D = pile diameter; H = diagonal of rectangular shape or HP pile.

The BOCA code also stipulates that spacing for friction piles in loose sand or loose sand-gravels shall be increased 10 percent for each interior pile to a maximum of 40 percent. Optimum spacing seems to be on the order of 2.5D to 3.5D or 2H to 3H for vertical loads; for groups carrying lateral and/ or dynamic loads, larger pile spacing are usually more efficient. Maximum pile spacing are not given in building codes, but spacing as high as 8D or 10D have been used on occasion.
EFFICIENCY OF PILE GROUP

The ultimate load capacity of pile group is defined as:

\[ Q_{P(group)} = \sum Q_{P(single)} \times \eta \]

where: \( \sum Q_P = Q_{P(single)} \times (N) \); \( N \) = No. of piles group, and \( \eta \) = efficiency of pile group.

(a) PILE GROUP EFFICIENCY IN CLAY

Several methods are available to calculate the efficiency of piles groups in clay; these are:

1. Convers-Labarre Method:

\[ \eta = 1 - \frac{\theta}{90} (n - 1)m + (m - 1)n \]

\[ \frac{1}{\eta} = 1 + \frac{N^2Q_p^2(single)}{Q_p^2(group)} \]

where:

- the parameters are as defined in Figure;
- \( \eta \) = efficiency,
- \( m \) = no. of rows,
- \( n \) = no. of piles in each row,
- \( \theta \) = \( \tan^{-1} \frac{d}{s} \),

where: \( d \) = diameter of pile, and
- \( s \) = spacing between piles.

2. Poules and Davis (1980) Method:

Note:- If \( \eta \geq 1.0 \); Use \( \eta = 1.0 \)

Allowable load capacity of pile group:

\[ Q_{all.(group)} = Q_{ult.(group)}/S.F. \]

where: 2.5 ≤ S.F. ≤ 4.5
(b) PILE GROUP EFFICIENCY IN SAND

Usually piles used for sand are driven piles, therefore, driving of the pile is a difficult process even though leads to densify the sand at deeper stratum.

The pile group efficiency in sand is calculated by the same equations that used for efficiency of piles groups in clay, but with $Q_{P\text{group}}$, ultimate load capacity of piles group, calculated according to Kezeli (1975) suggestion as follows:

1. If $S < d.K_p / K_o$ ............... The group action is control, and for this case:

   \[ Q_{P\text{group}} = 2(a + b).L.(k_s.\sigma'_v.\tan \phi_u) + a.b.q_{(base)} N_q \]

   where:
   - $S =$ spacing between the piles c/c,
   - $d =$ diameter of pile,
   - $K_p = \tan^2 (45 + \phi/2) = \frac{1 + \sin \phi}{1 - \sin \phi}$; the passive earth pressure coefficient,
   - $K_o = 1 - \sin \phi$; the at rest earth pressure coefficient;
   - Knowing that $K_a < K_o < K_p$, and $K_a = \frac{1}{K_p}$; the active earth pressure coefficient.

2. If $S > d.K_p / K_o$ ............... The single action is control, and for this case:

   \[ Q_{P\text{group}} = Q_{P\text{single}}(N) \]

   where:
   - $Q_{P\text{single}}$ is calculated from Broom’s or Nurdlund’s methods, and
   - $N =$ total number of piles.

NEGATIVE SKIN FRICTION

It is a side friction between the pile and the surrounding soil (fill or fill + compressible stratum) acting in a direction such that is to increase the load on the pile (Drag Force) and therefore should be subtracted from the design load.
NEGATIVE SKIN FRICTION ESTIMATION

1. Cohesive Soil:
   For this type of soil, the negative unit skin friction = $u_C \alpha$.
   $\alpha$ is obtained using Tomlinson’s method chart for driven piles,
   $\alpha = 0.45$ for bored piles,
   $u_C = u_C'$ of the compressible layer (case 2).

2. Cohesionless Soil:
   For this type of soil, the negative unit skin friction = $k_s \sigma' \tan \delta$.

Case (1): Fill over bearing incompressible stratum:

(a) **Normally placed piles (single action)**
   \[ Q_{neg.} = \pi.d.h.N.(k_s \sigma'_v \tan \delta); \]
   where: $N = \text{no. of piles}$.

(b) **Closely placed piles (group action)**
   \[ Q_{neg.} = Q_1.(\text{wt. of fill}) + Q_2.(\text{drag down force}) \]
   \[ = a.b.h.\gamma_{fill} + 2(a + b).h.(k_s \sigma'_v \tan \phi_{fill}) \]
   where: $a$ and $b = \text{short and long outside dimensions of the group}$, respectively.

Case (2): Fill over compressible stratum and bearing incompressible stratum:

(a) **Normally placed piles (single action)**
   \[ Q_{neg.} = \pi.d.h.N.(k_s \sigma'_{v(avg)} \tan \delta) + \pi.d.H.N.(\alpha.C_u) \]

(b) **Closely placed piles (group action)**
   \[ Q_{neg.} = a.b.h.\gamma_{fill} + a.b.H.\gamma_{clay} + 2(a + b).h.(k_s \sigma'_v \tan \phi_{fill}) + 2(a + b).H.(\alpha.C_{uclay}) \]
NOTE: From each case, for design, the lesser of the two values should be taken.

The values of $k_s$ and the unit negative skin friction in clay, can be determined from the following table as suggested by Bjerrum:

<table>
<thead>
<tr>
<th>Type of clay</th>
<th>$\phi'$</th>
<th>$k_s$</th>
<th>unit negative skin friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silt</td>
<td>30°</td>
<td>0.45</td>
<td>$0.25 \sigma'_v$</td>
</tr>
<tr>
<td>Low plasticity</td>
<td>20°</td>
<td>0.50</td>
<td>$0.20 \sigma'_v$</td>
</tr>
<tr>
<td>Plastic</td>
<td>15°</td>
<td>0.55</td>
<td>$0.15 \sigma'_v$</td>
</tr>
<tr>
<td>High plasticity</td>
<td>10°</td>
<td>0.60</td>
<td>$0.10 \sigma'_v$</td>
</tr>
</tbody>
</table>
When the pile group was subjected to moment, the reaction on each pile mainly due to two parts:

(a) Central axial load, and
(b) Moment.

Since, pressure is linearly distributed:

\[ \frac{R_1}{x_1} = \frac{R_2}{x_2} = \frac{R_3}{x_3} = \frac{R_4}{x_4} \]  
(By interpolation)

or

\[ R_2 = \frac{x_2 \cdot R_1}{x_1} \]
\[ R_3 = \frac{x_3 \cdot R_1}{x_1} \]
\[ R_4 = \frac{x_4 \cdot R_1}{x_1} \]  
……………………. (24)

Taking moment about the center of gravity of group:-

\[ \Sigma M_{uy} = 0; \quad \text{Applying moment = Resisting moment} \]
\[ \Sigma M_{uy} = x_1 \cdot R_1 + x_2 \cdot R_2 + x_3 \cdot R_3 + x_4 \cdot R_4 \]

Substituting \( R_2, R_3, R_4 \) from Eq.(11.28) gives:

\[ \Sigma M_{uy} = x_1 \cdot R_1 + x_2 \cdot \frac{x_2 \cdot R_1}{x_1} + x_3 \cdot \frac{x_3 \cdot R_1}{x_1} + x_4 \cdot \frac{x_4 \cdot R_1}{x_1} \]

\[ \Sigma M_{uy} = \frac{x_1^2 \cdot R_1}{x_1} + \frac{x_2^2 \cdot R_1}{x_1} + \frac{x_3^2 \cdot R_1}{x_1} + \frac{x_4^2 \cdot R_1}{x_1} \]

\[ \Sigma M_{uy} = \frac{R_1}{x_1} \left[ \frac{x_1^2}{x_1} + \frac{x_2^2}{x_2} + \frac{x_3^2}{x_3} + \frac{x_4^2}{x_4} + \ldots \right] = \frac{R_1}{x_1} \sum_{i=1}^{n} \frac{x_i^2}{x_i} \]

\[ \therefore \]  
Moment reaction on piles (1), (2), (3), and (4) can be determined, respectively as:
\[ R_1 = \frac{x_1 \cdot \sum Mu_{yy}}{n} \quad ; \quad R_2 = \frac{x_2 \cdot \sum Mu_{yy}}{\sum x_i^2} \quad ; \quad R_3 = \frac{x_3 \cdot \sum Mu_{yy}}{n} \quad ; \quad R_4 = \frac{x_4 \cdot \sum Mu_{yy}}{\sum x_i^2} \]

Similarly; **Moment reaction on pile (n):**

\[ R_n = \frac{x_n \cdot \sum Mu_{yy}}{\sum x_i^2} \quad \text{for moment in one-direction) } \quad \text{(25-a)} \]

Assuming a sign convention as: compression (+) positive and tension (−) negative, then the full reaction on each pile due to central axial load and moment will be written as:-

\[ P_1 = \frac{\sum Pu}{n} + R_1 \quad ; \quad P_2 = \frac{\sum Pu}{n} + R_2 \quad ; \quad P_3 = \frac{\sum Pu}{n} - R_3 \quad ; \quad P_4 = \frac{\sum Pu}{n} - R_4 \]

In general:

\[ P_n = \frac{\sum Pu}{n} + \frac{x_n \cdot \sum Mu_{yy}}{\sum x_i^2} \quad \text{for moments in two directions) } \quad \text{(25-b)} \]

where:

\[ \sum Pu = \text{total ultimate vertical load acting on pile,} \]
\[ n = \text{number of piles in the group,} \]
\[ x_n = x\text{-distance from center of the group (c.g.) to the pile (n) in question,} \]
\[ y_n = y\text{-distance from center of the group (c.g.) to the pile (n) in question,} \]
\[ \sum Mu_{yy} = \text{sum of ultimate moment in y-direction about center of gravity of group,} \]
\[ \sum Mu_{xx} = \text{sum of ultimate moment in x-direction about center of gravity of group,} \]
\[ \sum x_i^2 = \text{sum of squares of the x-distances to each pile from (c.g.) of the group, and} \]
\[ \sum y_i^2 = \text{sum of squares of the y-distances to each pile from (c.g.) of the group.} \]
DESIGN OF PILE CAP

A reinforced concrete slab which interconnects a group of piles and acts as a medium to transmit all the superstructure loads to the piles is called a **Pile Cap**. The loads may consist of vertical and horizontal loads, soil overlying the cap (if the cap is constructed below the ground surface), and moments, in addition to, the weight of the cap itself.

The pile cap should normally be rigid so as to distribute the loads equally on the piles of a group. In general it is designed like a footing on soil but with the difference that instead of uniform reaction from the soil, the reactions in this case are concentrated either point loads or distributed.

**Pile Layout Patterns**

Piles or other types of deep foundations under pile cap can layout symmetrically in both directions in several patterns as shown in Fig.(9). In general, the column or wall on the pile cap should be centered at its geometric center in order to transfer the load evenly to each pile.

![Diagram of Pile Layout Patterns](image)

*Fig.(9): Pile layout patterns.*
Pile Spacing, Edge Distance, and Pile Cap Thickness

➤ Pile Spacing

Spacing of piles depends upon the method of installing the piles and the type of soil. According to the building codes such as CP 2004, the minimum centre-to-centre spacing of piles should be taken as:

- For straight uniform diameter piles, 2.0d to 6.0d (where, d = pile diameter).
- For friction piles, 3.0d.
- For end bearing piles
  (i) passing through relatively compressible strata, the spacing of piles shall not be less than 2.5d
  (ii) For end bearing piles passing through compressible strata and resting in stiff clay, 3.5d
- For compaction piles, 2.0d

In general, piles should be spaced at 3d centre-to-centre in order to transfer load effectively to soil. If the spacing is $\leq 3d$, pile group settlement and bearing capacity should be checked.

<table>
<thead>
<tr>
<th>Pile diameter (mm)</th>
<th>300</th>
<th>350</th>
<th>400</th>
<th>450</th>
<th>500</th>
<th>550</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pile spacing (mm)</td>
<td>900</td>
<td>1050</td>
<td>1200</td>
<td>1350</td>
<td>1500</td>
<td>1650</td>
<td>1800</td>
</tr>
</tbody>
</table>

➤ Edge Distance of Piles

The edge distance is normally governed by punching shear capacity of corner piles.

➤ Pile Cap Thickness

Pile cap thickness is normally determined according to the shear strength requirements.

(i) For smaller pile cap, the thickness is governed by deep beam shear.
(ii) For large pile cap, the thickness is governed by wide-beam shear.
(iii) When necessary, shear reinforcement may be used for reducing thickness of pile cap.

Pile cap thickness is fixed such that it is adequate to resist shear without shear reinforcement and the bars projecting from the piles and the dowel bars for the column can be provided adequate bond length. For piles cap to be rigid, its minimum thickness should not be less than 600 mm. As a guide, the following formulae given for reinforced concrete may be used:

- For pile diameter (Dp) $\leq$ 550 mm: Pile cap thickness (h) $= (2 \, Dp + 100)$ mm
- For pile diameter (Dp) $>$ 550 mm: Pile cap thickness (h) $= (8 \, Dp + 600)/3$ mm.
**Practical Aspects on Pile Cap Design**

1. Pile cap should be perfectly rigid. In addition to, it should be deep enough to allow the necessary overlap of reinforcements from column and piles.
2. The span to thickness ratio of the cap should not be more than 5.0 so that pile cap is rigid enough to distribute the load uniformly to all piles.
3. Since the piles are short and elastic columns, the deformations and stress distribution are planer.
4. Pile heads are hinged to the pile cap and hence no bending moment is transmitted to piles from pile caps.
5. Pile heads should be embedded at least (150 – 300) mm into the cap. In addition, the bottom rebars should loop around the pile to avoid splitting a part of the cap from pile head moments and shears.
6. For accommodating deviations in driving of piles, pile cap should be extended at least (150 – 300) mm beyond the outside faces of exterior piles (i.e., clear overhang beyond the outermost pile not less than 150mm).
7. Pile cap should be reinforced for both positive and negative bending moments. The bottom cap reinforcing bars should be 7.5cm above piles heads to control concrete cracking around them as shown in Fig.(10).
8. The minimum effective depth of the pile cap is (d = 300 mm); (as required by ACI–318 Code—in Art. 15–7). Therefore, referring to Fig.(10), the minimum cap thickness is: \( t = 300 + \text{d}_{\text{bar}} / 2 + 75 + 150 \) (mm).
9. Tension shear connectors should be used on the pile heads if the piles are subjected to tension forces.
10. The critical sections for piles cap shear and moment are computed in the same way as that of spread footings taking into account the criteria shown in Fig.(11).

![Fig.(10): Design of pile cap.](image-url)
Design Procedure of Pile Cap

1. Estimate number of piles needed.
2. Select pile layout pattern.
3. Convert the loads into ultimate.

where, \( P_u = 1.2 \, DL + 1.6LL \)

4. Calculate individual pile loads or reactions:
   (a) For concentric loaded pile cap (eccentricity = 0); each pile carries an equal amount of the ultimate load and for \( n \) piles carrying a total load \( \sum P_u \), the load per pile is:

   \[ P_n = \frac{\sum P_u}{n} \]

   This assumption is correct when all the piles are vertical, the pile cap is in contact with the ground surface, and the piles cap is rigid.

   (b) For eccentric loaded pile cap (eccentricity \( \neq 0 \)) in two directions; each pile carries certain value due to load and moment as:
\[
P_n = \frac{\sum P_u}{n} \pm \frac{x_n \cdot \sum M_{uy}}{\sum x_i^2 i=1} \pm \frac{y_n \cdot \sum M_{ux}}{\sum y_i^2 i=1}
\]

where,

\(P_n\) = pile load or reaction.
\(\sum P_u\) = total ultimate load,
\(\sum M_{ux}, \sum M_{uy}\) = ultimate moments about x and y axes, respectively,
\(x, y\) = distances from y and x axes to any pile,
\(\sum x_i^2, \sum y_i^2\) = moment of inertia of the group, computed as: \(I = I_o + A.d^2\) but the pile moment of inertia \(I_o\) is negligible, thus the \(A\) term cancels,

\textit{Notice that the maximum pile load shall not exceed allowable pile capacity.}

5. Find pile cap thickness:

Calculate factored shear at critical sections. The one-way or (wide beam shear) is checked at a distance of \(d\) from the face of the column. The critical section for two-way shear (punching shear) is at a distance \(d/2\) from face of column or pedestal.

\textit{In computing the external shear on any section,}

- The entire (100\%) reaction of any pile of diameter \(D_p\) whose centre is located \(D_p/2\) or more outside the section shall be taken.
- The pile will produce no shear (0\%) if the pile centre is located \(D_p/2\) or more inside the section.
- For intermediate positions of the pile centre, the pile reaction shall be based on straight line interpolation between full value at \(D_p/2\) outside the section and zero value at \(D_p/2\) inside the section.

6. Find pile cap reinforcement:

Pile cap has to be designed either by truss theory or beam theory. Although, the pile caps are assumed to act as a simply supported beam and are designed for the usual condition of bending and shear, their tendency is to fail by bursting due to high principal tension and they will therefore always require a cage of reinforcement in three dimensions to resist this tendency.

The critical section for bending moments and bond shall be calculated at the face of column or pedestal. The main reinforcement is usually bended and extended for full depth of pile cap to fulfill the development length check. For bursting (horizontal binders) it is suggested that 25 \% of the main reinforcement (usually 12 \(\phi\) mm at 150 mm c/c) shall be used. A cover of 75 mm is usually provided for the pile cap in contact with earth and 60 mm against blinding concrete of 75 to 100 mm thick. In marine situations the cover should be increased to a minimum of 80 mm.